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VISCOUS EFFECTS AND HEAT TRANSFER IN SEPARATED FLOW

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Boundary layer separation (and reattachment) is one of the oldest unsolved problems in fluid mechanics. When the flow is two-dimensional and the "outer" inviscid flow velocity is supersonic the problem is somewhat simplified, because the static pressure and the flow inclination angle induced by boundary layer growth are connected by the local Prandtl-Meyer relation. However, the question of the proper treatment of the viscous layer was still open when this study began. Because of the complexity of the governing differential equations one turns naturally to integral or moment methods in attacking the viscous layer. The well-known Kármán-Pohlhausen momentum integral method suffers from the defect that the velocity profile is uniquely determined by a single pressure gradient parameter $\Lambda(x) = (\theta^2/\nu_e)(du_e/dx)$, where θ is momentum thickness. In the laminar boundary layer-shock wave interaction problem, for example (Figure 1), the static pressure on the surface reaches a "plateau" upstream of shock impingement, but the velocity profile is certainly far from the Blasius profile corresponding to $\Lambda \longrightarrow 0$.

In an attempt to eliminate the shortcomings of the Kármán-Pohlhausen method Crocco[1] and Lees introduced a "mixing theory", in which the velocity profile is specified in terms of a single shape parameter $\mathcal{X}(x)$ not directly related to the local static pressure gradient. Integral properties are determined in terms of $\mathcal{X}$ by utilizing the Falkner-Skan and other similar-flow solutions. The price one pays for this freedom is the introduction of a mixing or mass entrainment rate parameter, which is specified in terms of $\mathcal{X}$ for attached flow, but is admittedly semi-empirical in separated and reattaching flows.

The main objective of the present investigation is to eliminate the semi-empirical features of the Crocco-Lees method, while retaining the flexibility gained by "unhooking" the velocity profile from the local pressure gradient. A second objective is to develop a method that is sufficiently general to include heat transfer (the original Crocco-Lees theory is limited to adiabatic flow). In our present approach we have generalized and extended Tani's[2] method to apply to separated and reattaching flows[3]. Tani utilizes not only the momentum integral, but also the first moment of momentum integral across the viscous layer.* In this method the velocity profiles are specified by a single parameter a(x), which corresponds to the slope of the profile at the surface in attached flow, and to the relative distance of the zero velocity line above the surface in separated and reattaching flow. The viscous layer is governed by the two functions $a(x)$ and $\theta(x)$. In addition we employ the continuity equation, integrated outward from the surface to obtain the streamline inclination of the "outer" inviscid flow, and the Prandtl-Meyer relation connecting the pressure and flow inclination in the outer flow. These simultaneous first-order non-linear differential equations are programmed for solution on an IBM 7090 Computer.

At first we employed the Tani quartic for the velocity profile. The results of our calculations showed good agreement with experimental surface pressure data on laminar boundary layer-shock wave interaction, provided the impinging shock is relatively weak. But for stronger shocks the results show a smooth transition from attached flow to a pressure maximum, and then the pressure gradually decreases toward the shock impingement point (Figure 2). In order to overcome this difficulty we abandoned the quartic and replaced it with the reversed-flow velocity profiles of Stewartson[4], corresponding to the "lower branch" of the Falkner-Skan solutions. Of course it was essential to replace Stewartson's β by our free parameter a(x). This family of profiles has the correct qualitative properties in separated and reattaching flows, especially the almost-linear movement of the dividing streamline away from the surface with downstream distance, and the later return to the surface downstream of shock impingement. For adiabatic flow a comparison between our theoretical calculations and the experiments of Chapman[5] and Hakkinen[6] show good agreement (Figures 2 and 3). This method can also be applied to supersonic flow in a corner, flow over flaps and flares, etc. This work is continuing under Grant AF-AFOSR-54-63, and a report by B. L. Reeves and L. Lees is in preparation.

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* Evidently any number of moments could be employed; each additional moment introduces an additional first-order ordinary differential equation and makes possible an additional degree of freedom in the velocity profiles.

As a first step in a study of viscous flows with heat transfer Stuart Savage reexamined the Tani method for attached boundary layers. Most investigators have employed an approximate enthalpy-velocity relation, which is somewhat dubious. Savage[7] found that a unique stagnation enthalpy profile can be utilized with good accuracy upstream of separation. With the aid of this device he solved the well-known Howarth non-similar flow problem with a prescribed linearly decreasing external velocity, and found excellent agreement with "exact" numerical solutions of the boundary layer differential equations.

Savage's idea has now been extended to solve the laminar boundary layer-shock wave interaction problem for cooled surfaces, except that we utilize the Cohen-Reshotko similar solutions for heat transfer along both upper and lower branches*, instead of Stewartson's adiabatic flow solutions. In other words the total enthalpy profile is characterized completely by the parameter a(x) and the ratio of surface enthalpy to free-stream total enthalpy. The results of these solutions show that for a given Mach number, Reynolds number at separation and incident shock strength, the length of separated flow decreases markedly as cooling is increased. This result is in qualitative agreement with some experiments at JPL.

For highly cooled surfaces our theoretical results show certain anomalies upstream of separation which are probably caused by the over-simplification inherent in a one parameter family of velocity profiles. Therefore we propose to investigate a two-parameter family of velocity profiles within the framework of the two-moment integral approach. The second parameter, in addition to a(x), might be the pressure gradient parameter $\Lambda(x)$; in other words both the slope and curvature of the velocity profile are taken to be undetermined functions of x. The additional relation required might be obtained by satisfying the boundary layer equation along the surface.

For cooled surfaces we also discovered one important effect that we believe to be real. At a certain critical surface-to-free stream

* Along the upper branch $u > 0$ everywhere, while along the lower branch regions of reversed flow exist. These two branches join smoothly at the separation (or reattachment) profile.

temperature ratio (depending on Mach number) a smooth approach to separation from the Blasius flow is no longer possible. In other words the laminar boundary layer becomes "super-critical" just as the adiabatic turbulent layer does in the Crocco-Lees mixing theory. This behavior means that the laminar boundary layer-shock wave interaction on a highly cooled surface is qualitatively different from the usual phenomenon observed on adiabatic surfaces. Presumably the flow exhibits a "shock", which brings the boundary layer to the sub-critical state in a distance of a few boundary layer thicknesses, after which the flow proceeds through separation as before.

These studies are also continuing under Grant AF-AFOSR-54-63.

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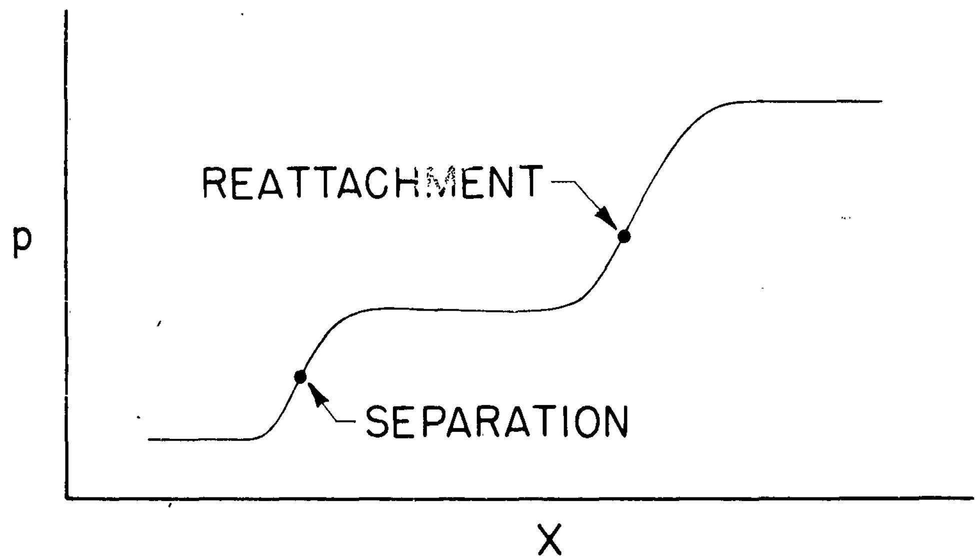

FIGURE 1

TYPICAL STATIC PRESSURE VARIATION ON A FLAT PLATE
IN SUPERSONIC FLOW WITH SEPARATION
INDUCED BY AN INCIDENT SHOCK WAVE

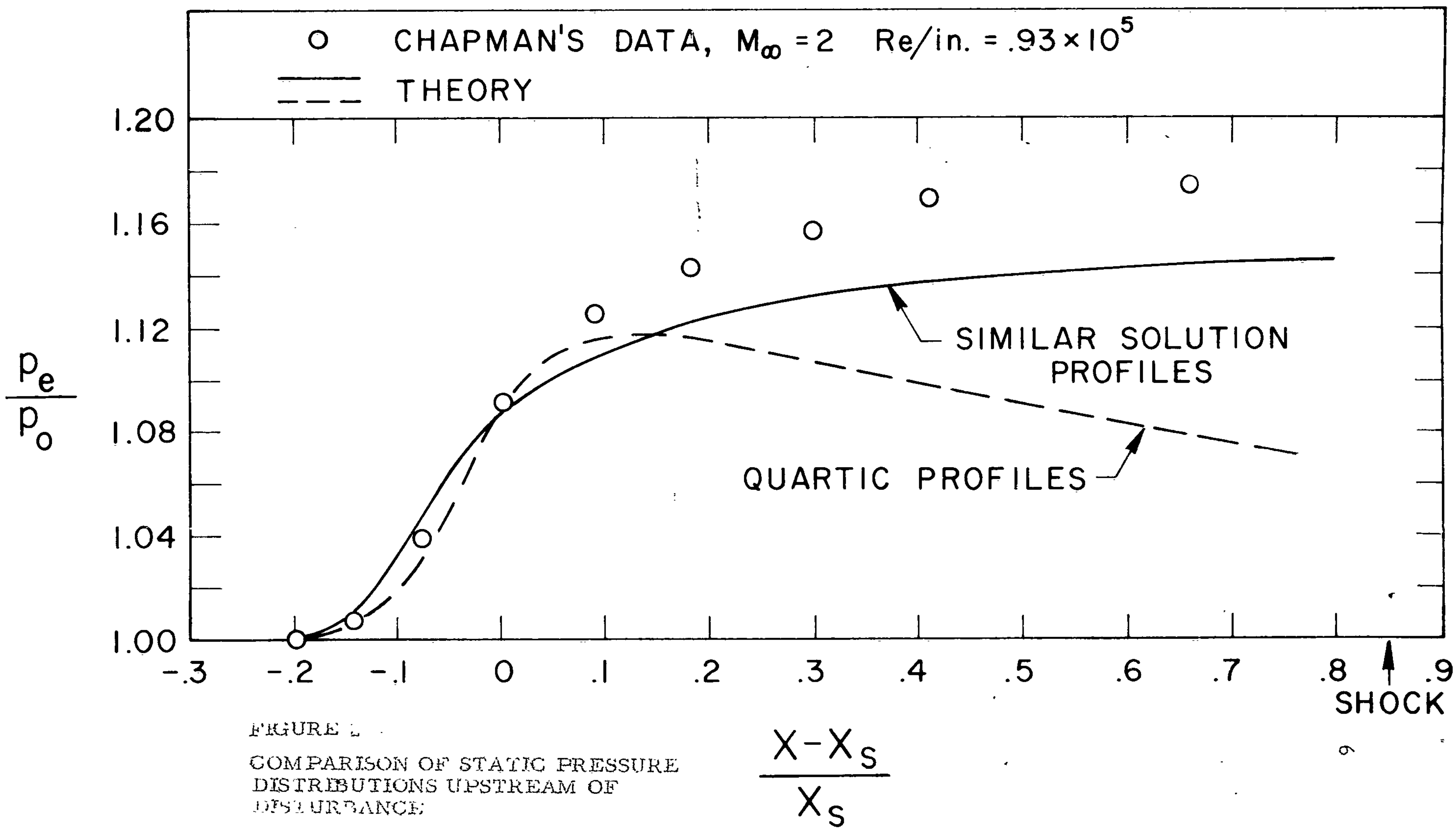

FIGURE 2

COMPARISON OF STATIC PRESSURE DISTRIBUTIONS UPSTREAM OF DISTURBANCE

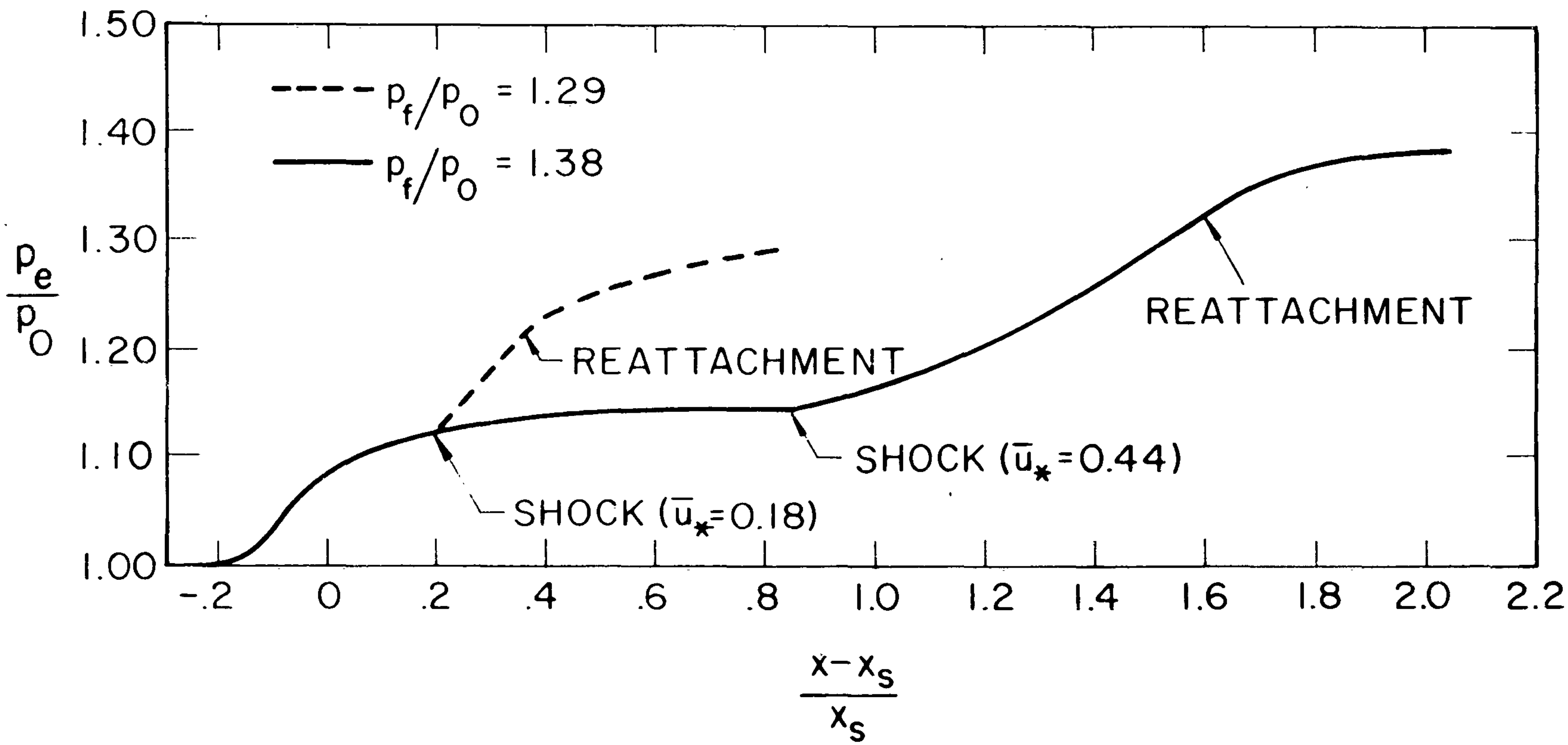

FIGURE 3

THEORETICAL PRESSURE DISTRIBUTIONS AT M_∞ = 2.0 Re /inch = 1.86 x 10^5